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# FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE SHALLOW SHELLS

**SUNU MARIAM SAJI[1] & PRABHA C[2]**

[1]Computer Aided Structural Engineering, Mar Athanasius College of Engineering, Kothamangalam, Kerala, India

[2]Assistant Professor, Department of Civil Engineering, Mar Athanasius College of Engineering, Kothamangalam, Kerala, India

## ABSTRACT

Shell structures constituted a major component of today's aerospace, submarine, automotive and other machine or structural elements. They were used in aircraft, spacecraft, rockets, cars, computers, submarines, boats, storage tanks and the roofs of buildings. With the progress of composite materials, shallow shells constructed by composite laminas were extensively used in many fields of modern engineering practices. If external vibrations coincided with natural frequency of the material, resonance and subsequent failure result. Therefore free vibrations leading to the notions of the natural vibration frequency was to be studied.

This thesis work focused on the free vibrations of shallow thin composite shells of various shapes, modulus ratios, span-to-thickness ratios, boundary conditions and lay-up sequences. Most of the previous studies on this subject were confined to the classical boundary conditions. So the effect of elastically restrained edges on various shell curvatures including spherical and cylindrical shell was also studied.

**KEYWORDS:** Spherical Shell, Cylindrical Shell, Normalised Frequency Parameter, Natural Frequency

## INTRODUCTION

### General

Shell structures constitute a major component of today's aerospace, submarine, automotive and other machine or structural elements. They can be used for aerodynamic, aesthetic and/or other reasons. They are used in aircraft, spacecraft, rockets, cars, computers, submarines, boats, storage tanks and the roofs of buildings. In the last decade, the continuous development of material science and engineering, along with the increasing demand to produce light-weight structures, has led to the use of advanced materials (laminated composites and functionally graded materials (FGMs)) in designing shell structures. Composite laminates are widely used in demanding applications, as for example in aerospace structures, where large dynamic loads occur and the temperature varies.

With the progress of composite materials, shallow shells constructed by composite laminas are extensively used in many fields of modern engineering practices requiring high strength-weight and stiffness-weight ratios such as aircraft structures, space vehicles and deep-sea engineering equipments. Shallow shells are open shells that have small curvatures (i.e. large radii of curvatures compared with other shell parameters such as length and width). They can have circular, rectangular, triangular or any other planform. They can be singly-curved (i.e. cylindrical) or doubly curved (e.g. spherical). They can also have their principal radii of curvature not align with the geometric boundaries, introducing a radius of twist (e.g. turbo machinery blades).

**Objectives of Thesis Work**

- Study of free vibrations of shallow thin composite shells of various shapes, modulus ratios, span-to-thickness ratios, boundary conditions and lay-up sequences.
- Study the effect of boundary conditions on dynamic characteristics of composite shallow shells.
- Study the influence of curvature, panel thickness and fibre orientation on the shells' dynamics.

## FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE SHALLOW SHELL

### General

From the practical point of view, it is essential to have knowledge of the free vibration characteristics of shells in structures required for space missions. The destructive effect of resonance may be avoided if we know beforehand the fundamental frequencies of the structures. Finite element modelling is an art and is not governed by hard and fast rules. The results obtained from finite element method are approximate to the physical system, which is represented by a discrete mathematical model. The physical system has infinite degrees of freedom whereas the finite element model has finite numbers of degrees of freedom. The degree of approximation in the calculated results depends on how closely the physical system is modelled. The modelling process involves many possibilities – the element shape and types, the number of elements and boundary conditions. Error in each of these factors may cause the misrepresentation of the actual physical system.

ANSYS is a finite element analysis (FEA) software package. It uses a pre-processor software engine to create geometry. Then it uses a solution routine to apply loads to the meshed geometry. Finally it outputs desired results in post-processing. The subspace iteration procedure is used to solve for the natural frequency and eigen vectors.

In order to analyse the model in ANSYS, SHELL281 element is selected. The thickness for each ply and its orientations are defined. Then the orthotropic material property and density for analysis are defined. In order to model cylindrical structure – CSYS, 1 and spherical structure – CSYS, 2 are used. Then the model is meshed to provide accurate numerical solution. Then the specific boundary conditions are applied on each side. Model is analysed. After solving, we get natural frequency in 'Hertz' and its mode shapes. Material taken for analysis is free graphite/epoxy shell. The properties of shell considered are:

**Table 1: Material Properties of Shell**

| Property | Value |
|---|---|
| Young's Modulus-Longitudinal, $E_x$ | 138GPa |
| Young's Modulus-Transverse, $E_y$ | 8.96GPa |
| Shear Modulus, G | 7.1GPa |
| Poisson's Ratio, μ | 0.3 |
| Mass Density, ρ | $1500kg/m^3$ |

## EFFECT OF PARAMETERS ON NATURAL FREQUENCY

### Effect of Lay-Up Sequence on Natural Frequency

To study the effect of ply orintation, kept the value of a/b=1, $R_x/a=2$, h/a=1/100 and the frequencies are expressed in non dimensional parameter as $\Omega = \omega a^2\sqrt{(\rho/E_x h^2)}$.

### Spherical Shell

**Table 2: Normalised Frequency Parameter with Clamped Condition**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 32.944 | 33.334 | 33.621 | 34.007 | 41.964 | 43.363 | 44.307 | 45.250 |
| 0/45/0 | 27.731 | 31.819 | 35.615 | 36.975 | 40.317 | 6.5305 | 43.794 | 44.461 |
| 0/90/90/0 | 32.368 | 33.440 | 33.666 | 35.113 | 40.464 | 41.867 | 42.822 | 43.399 |
| 0/90 | 28.698 | 30.312 | 30.708 | 32.217 | 37.217 | 37.914 | 38.739 | 39.926 |
| 90/45/-45/90 | 31.460 | 36.032 | 37.696 | 37.984 | 40.095 | 41.502 | 43.333 | 45.453 |
| 0/90/45/-45/0/90 | 35.788 | 36.542 | 37.429 | 38.250 | 42.686 | 44.108 | 45.480 | 48.297 |

**Table 3: Normalised Frequency Parameter with Simply Supported Condition**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 30.515 | 31.506 | 32.841 | 33.400 | 38.359 | 40.773 | 41.316 | 42.306 |
| 0/45/0 | 24.515 | 28.633 | 32.545 | 33.497 | 38.235 | 38.676 | 42.541 | 42.626 |
| 0/90/90/0 | 29.526 | 30.342 | 32.731 | 33.663 | 38.084 | 38.395 | 38.895 | 40.759 |
| 0/90 | 27.262 | 27.393 | 29.961 | 30.119 | 34.325 | 34.910 | 36.415 | 37.269 |
| 90/45/-45/90 | 29.511 | 32.496 | 36.331 | 36.533 | 39.278 | 39.780 | 41.901 | 42.876 |
| 0/90/45/-45/0/90 | 34.627 | 35.547 | 35.694 | 36.765 | 39.571 | 40.895 | 41.565 | 44.306 |

For clamped condition, [0/90/45/-45/0/90] spherical shallow shell may be thought to be the stiffest in this study. For example, the first mode of the spherical shallow shell with [0/90/45/-45/0/90] is 29.1% greater than shell with [0/45/0]. In this case [0/90] is the most flexible one except the first mode of the spherical shell. It can be seen that first, fifth, sixth and seventh modal frequencies of [45/-45/45/-45] shell is higher than [90/45/-45/90]. The lowest first frequency exist in [0/45/0] ply layout sequence.

Also for simply supported condition, [0/90/45/-45/0/90] spherical shallow shell is the stiffest one. So for spherical shallow shell with given condition, [0/90/45/-45/0/90] lay-up sequence is the best one. Except the first mode, we can see from Table 3 that the [0/90] lay-up is the flexible one. By comparing the spherical shallow shell with clamped condition and simply supported condition, clamped one gives good results than second one .

### Cylindrical Shell

**Table 4: Normalised Frequency Parameter with Clamped Condition**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 18.968 | 20.453 | 25.783 | 27.334 | 28.315 | 31.312 | 33.170 | 34.766 |
| 0/45/0 | 20.743 | 26.677 | 29.417 | 31.060 | 33.943 | 35.475 | 39.751 | 40.503 |
| 0/90/90/0 | 18.990 | 24.455 | 26.419 | 28.905 | 31.190 | 33.437 | 39.109 | 39.435 |
| 0/90 | 12.655 | 16.382 | 19.916 | 21.025 | 28.031 | 28.421 | 28.517 | 32.317 |
| 90/45/-45/90 | 14.034 | 14.165 | 20.468 | 21.033 | 24.659 | 24.771 | 26.923 | 27.963 |
| 0/90/45/-45/0/90 | 18.157 | 22.735 | 26.827 | 28.507 | 33.911 | 36.315 | 37.089 | 37.820 |

**Table 5: Normalised Frequency Parameter with Simply Supported Condition**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 17.761 | 17.805 | 24.683 | 26.022 | 26.499 | 28.541 | 28.720 | 29.228 |
| 0/45/0 | 16.923 | 23.528 | 25.172 | 27.196 | 31.328 | 32.141 | 37.113 | 37.810 |
| 0/90/90/0 | 14.310 | 20.840 | 22.663 | 25.280 | 27.531 | 29.861 | 34.957 | 35.977 |
| 0/90 | 9.971 | 13.055 | 17.227 | 17.250 | 22.396 | 24.002 | 24.724 | 25.350 |
| 90/45/-45/90 | 11.558 | 12.760 | 16.964 | 20.103 | 20.425 | 20.791 | 23.444 | 23.819 |
| 0/90/45/-45/0/90 | 15.241 | 18.755 | 23.781 | 24.078 | 32.285 | 32.323 | 32.688 | 33.045 |

In cylindrical shell with clamped condition, the lay-up sequence [0/45/0] shows the critical condition. Then [0/90/90/0] lay-up sequence is the second one. In this case [0/90/45/-45/0/90] might not be the stiffest one when compared to [0/45/0] lay-up sequence. Except the first 3 modes, the flexible lay-up is [90/45/-45/90]. By comparing Table 3 and Table 4 it can be concluded that depending upon their curvatures variations, lay-up sequences which gives stiffness may supposed to change. In cylindrical shell with simply supported condition, only the first mode of [45/-45/45/-45] lay-up sequence is 4.95% higher than the [0/45/0] lay-up sequence. After the first mode, [0/45/0] lay-up sequence is supposed to be stiffer than all other lay-up sequence. From the Table 5 we can see that except the first mode [90/45/-45/90] lay-up sequence is the flexible one. In this case also by comparing the spherical shallow shell with clamped condition and simply supported condition, clamped one gives good results than second one.

**Effect of Thickness on Natural Frequency**

The natural frequency of shell for h/a=1/100 was studied in the previous chapter. To know the effect of frequency with the variation of thickness, h/a=1/50 ratio was taken for analysis, where the value of 'a' kept as constant. That is thickness is greater than previous one. Also kept the value of a/b=1, $R_x$/a=2 and the frequencies were expressed in non dimensional parameter as $\Omega=\omega a^2\sqrt{(\rho/E_x h^2)}$. In the equation 'h' is in denominator. If frequency parameter gets increase, natural frequency will decreases with thickness otherwise, it will increase with thickness.

**Spherical Shell**

**Table 6: Normalised Frequency Parameter with Clamped Condition for h/a=1/50**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 20.365 | 20.472 | 20.549 | 24.798 | 27.086 | 28.230 | 33.519 | 33.885 |
| 0/45/0 | 20.217 | 21.376 | 22.668 | 23.675 | 25.164 | 26.906 | 28.180 | 32.236 |
| 0/90/90/0 | 19.320 | 21.293 | 23.221 | 23.906 | 25.485 | 28.472 | 30.401 | 34.711 |
| 0/90 | 18.449 | 18.503 | 18.958 | 20.329 | 24.531 | 24.949 | 25.752 | 26.509 |
| 90/45/-45/90 | 21.160 | 21.329 | 21.419 | 23.723 | 24.886 | 27.019 | 28.597 | 32.471 |
| 0/90/45/-45/0/90 | 21.431 | 21.974 | 22.222 | 25.177 | 30.045 | 32.168 | 32.962 | 34.443 |

**Table 7: Normalised Frequency Parameter with Simply Supported Condition for h/a=1/50**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 18.517 | 19.627 | 19.731 | 20.960 | 24.939 | 26.084 | 29.137 | 29.491 |
| 0/45/0 | 16.121 | 19.216 | 21.455 | 22.630 | 22.938 | 23.706 | 25.166 | 27.586 |
| 0/90/90/0 | 18.166 | 18.796 | 19.766 | 20.124 | 23.546 | 24.476 | 26.380 | 28.600 |
| 0/90 | 16.197 | 16.399 | 16.941 | 18.523 | 21.009 | 21.540 | 22.809 | 24.713 |
| 90/45/-45/90 | 17.520 | 20.585 | 20.847 | 21.918 | 22.825 | 24.011 | 25.609 | 28.203 |
| 0/90/45/-45/0/90 | 19.226 | 19.925 | 21.299 | 21.886 | 25.729 | 27.333 | 27.562 | 28.578 |

In Table 6, [0/90/45/-45/0/90] lay-up sequence is 16.2% stiffer than [0/90]. Comparing Table 2 and Table 6 frequency parameter gets decreases. That is natural frequency increases with thickness.

**Cylindrical Shell**

**Table 8: Normalised Frequency Parameter with Clamped Condition for h/a=1/50**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 13.112 | 14.199 | 16.665 | 21.564 | 21.773 | 23.830 | 29.130 | 30.263 |
| 0/45/0 | 17.980 | 18.815 | 20.034 | 21.018 | 22.477 | 25.058 | 26.325 | 30.068 |
| 0/90/90/0 | 16.311 | 17.107 | 18.252 | 20.247 | 22.693 | 25.513 | 30.025 | 32.297 |

**Table 8: Contd.,**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0/90 | 10.304 | 13.776 | 15.208 | 16.838 | 20.003 | 22.384 | 22.407 | 22.597 |
| 90/45/-45/90 | 10.440 | 10.975 | 14.050 | 18.888 | 19.386 | 20.100 | 22.538 | 26.246 |
| 0/90/45/-45/0/90 | 14.782 | 15.216 | 19.609 | 20.900 | 25.874 | 28.639 | 29.554 | 30.533 |

**Table 9: Normalised Frequency Parameter with Simply Supported Condition for h/a=1/50**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 11.708 | 12.720 | 14.650 | 17.497 | 18.382 | 19.969 | 24.414 | 25.926 |
| 0/45/0 | 12.865 | 16.535 | 18.837 | 19.421 | 20.953 | 21.251 | 24.197 | 25.554 |
| 0/90/90/0 | 11.781 | 15.147 | 16.071 | 17.177 | 20.185 | 20.235 | 24.890 | 26.086 |
| 0/90 | 7.028 | 11.362 | 12.714 | 14.838 | 15.495 | 16.259 | 17.890 | 19.871 |
| 90/45/-45/90 | 8.206 | 9.049 | 11.614 | 14.835 | 15.140 | 15.381 | 17.381 | 21.468 |
| 0/90/45/-45/0/90 | 10.574 | 14.636 | 16.022 | 17.111 | 19.647 | 22.499 | 23.756 | 24.287 |

In Table 8, the first mode frequency of [0/45/0] lay-up sequence is 42.7% greater than [0/90]. In Table 9, it is 45.4%. When the value of thickness increases since ‘h’ is in denominator value of normalised frequency parameter gets decrease. In fact the natural frequency keeps on increasing, when thickness increases. For any type of curvature when thickness increases, natural frequency increases.

**Effect of Radius of Curvature on Natural Frequency**

The natural frequency of shell for $R_x/a=2$ was studied in previous chapter. In this chapter to know the variation of natural frequency with radius of curvature, $R_x/a=3$ was taken, where ‘a’ is same as above. Here radius of curvature is greater than previous one. Also kept the value of a/b=1, h/a=1/100 and the frequencies were expressed in non dimensional parameter as $\Omega=\omega a^2\sqrt{(\rho/E_x h^2)}$.

Comparing the highest frequencies of Table 2 and Table 10, [0/90/45/-45/0/90] lay-up sequence of $R_x/a=2$ is 22.4% stiffer than $R_x/a=3$. Also for simply supported condition, comparing the highest frequencies of Table 3 and Table 11, [0/90/45/-45/0/90] lay-up sequence of $R_x/a=2$ is 23% stiffer than $R_x/a=3$.

**Spherical Shell**

**Table 10: Normalised Frequency Parameter with Clamped Condition for $R_x/a=3$**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 24.890 | 25.026 | 25.133 | 27.483 | 31.838 | 33.645 | 37.310 | 37.465 |
| 0/45/0 | 22.750 | 25.458 | 27.648 | 28.864 | 29.925 | 31.300 | 32.868 | 35.746 |
| 0/90/90/0 | 24.088 | 26.014 | 26.154 | 27.078 | 31.601 | 31.724 | 34.224 | 37.269 |
| 0/90 | 21.672 | 22.862 | 23.016 | 24.882 | 28.658 | 28.882 | 29.132 | 31.338 |
| 90/45/-45/90 | 24.636 | 27.138 | 27.147 | 29.201 | 29.564 | 31.642 | 33.080 | 36.352 |
| 0/90/45/-45/0/90 | 26.159 | 26.803 | 27.743 | 29.054 | 34.139 | 36.462 | 36.962 | 37.474 |

**Table 11: Normalised Frequency Parameter with Simply Supported Condition for $R_x/a=3$**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 23.696 | 23.831 | 23.918 | 24.478 | 30.382 | 32.086 | 32.957 | 33.265 |
| 0/45/0 | 18.797 | 22.619 | 25.862 | 26.927 | 28.795 | 29.261 | 31.016 | 31.870 |
| 0/90/90/0 | 22.095 | 23.301 | 23.443 | 24.966 | 28.050 | 30.574 | 31.092 | 31.487 |
| 0/90 | 13.770 | 20.113 | 22.342 | 23.357 | 24.486 | 24.676 | 27.285 | 29.775 |
| 90/45/-45/90 | 21.420 | 25.812 | 26.704 | 26.848 | 28.098 | 29.154 | 30.763 | 32.493 |
| 0/90/45/-45/0/90 | 24.524 | 25.183 | 26.036 | 27.013 | 29.995 | 31.418 | 32.159 | 34.110 |

### Cylindrical Shell

**Table 12: Normalised Frequency Parameter with Clamped Condition for $R_x/a=3$**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 15.825 | 16.288 | 19.609 | 23.279 | 23.941 | 26.353 | 31.145 | 32.932 |
| 0/45/0 | 19.221 | 22.814 | 24.002 | 24.874 | 27.480 | 28.035 | 31.247 | 33.323 |
| 0/90/90/0 | 18.020 | 20.475 | 21.822 | 22.365 | 26.546 | 27.580 | 33.609 | 34.668 |
| 0/90 | 11.070 | 15.362 | 16.748 | 19.077 | 23.913 | 24.329 | 25.463 | 25.719 |
| 90/45/-45/90 | 11.702 | 12.496 | 15.978 | 19.600 | 21.530 | 21.810 | 23.989 | 27.221 |
| 0/90/45/-45/0/90 | 15.956 | 18.747 | 22.909 | 23.150 | 27.965 | 31.566 | 32.005 | 33.006 |

For cylindrical shell, comparing the highest frequencies of Table 4 and Table 12, [0/45/0] lay-up sequence of $R_x/a=2$ is 17.7% stiffer than $R_x/a=3$. Also for simply supported condition, comparing the highest frequencies of Table 5 and Table 13, [0/45/0] lay-up sequence of $R_x/a=2$ is 21.8% stiffer than $R_x/a=3$.

**Table 13: Normalised Frequency Parameter with Simply Supported Condition for $R_x/a=3$**

| Ply Layout | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| 45/-45/45/-45 | 13.775 | 15.501 | 17.953 | 19.637 | 21.055 | 22.387 | 26.299 | 27.928 |
| 0/45/0 | 14.144 | 19.103 | 22.465 | 23.307 | 24.927 | 26.350 | 29.389 | 29.576 |
| 0/90/90/0 | 12.654 | 16.995 | 19.921 | 21.345 | 22.565 | 24.451 | 27.342 | 28.633 |
| 0/90 | 7.885 | 12.868 | 13.133 | 16.220 | 19.948 | 20.313 | 21.272 | 22.328 |
| 90/45/-45/90 | 9.844 | 10.032 | 14.550 | 15.617 | 17.206 | 17.234 | 18.898 | 22.389 |
| 0/90/45/-45/0/90 | 12.046 | 17.294 | 18.496 | 20.745 | 22.744 | 24.718 | 26.877 | 27.373 |

Thus, it can be seen that when radius of curvature increases normalised frequency parameter get decreases. When radius of curvature increases without changing panel dimension, it becomes flatter than above. That is its structural behaviour will more similar to plate. From this we can conclude that the natural frequency gets decrease, when radius of curvature increases.

## EFFECT OF ELASTICALLY RESTRAINED EDGES ON NATURAL FREQUENCY

Elastically restrained laminated composite shallow shells are widely encountered in many engineering practices. The vibration analyses of these shells with such boundary conditions are necessary and of great significance. Thus in this part, the present formulations are applied to investigate the free vibration behaviors of laminated composite shallow shells with elastic supports. Elastically restrained edges are given by $E_1$ and $E_2$. $E_1$ specifies normal, tangential, and transverse directions are elastically restrained. $E_2$ specifies normal, tangential, transverse and rotation directions are elastically restrained. Parameters considered are a/b=1, h/a=1/100, $R_x/a=2$ and the frequencies were expressed in non dimensional parameter as $\Omega=\omega a^2\sqrt{(\rho/E_x h^2)}$.

### Spherical Shell

**Table 14: Spherical Shell with Elastically Restrained Edges of [0/90/45/-45/0/90] Lay-Up Sequence**

| | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| $E_1$-$E_1$-$E_1$-$E_1$ | 9.942 | 10.905 | 13.273 | 14.391 | 14.894 | 15.365 | 16.130 | 16.398 |
| $E_2$-$E_2$-$E_2$-$E_2$ | 12.639 | 14.845 | 15.597 | 17.118 | 20.385 | 21.775 | 24.055 | 29.309 |

**Cylindrical Shell**

**Table 15: Cylindrical Shell with Elastically Restrained Edges of [0/45/0] Lay-Up Sequence**

| | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
|---|---|---|---|---|---|---|---|---|
| $E_1$-$E_1$-$E_1$-$E_1$ | 2.763 | 5.282 | 7.342 | 8.526 | 9.239 | 10.883 | 13.096 | 14.136 |
| $E_2$-$E_2$-$E_2$-$E_2$ | 8.060 | 9.989 | 12.562 | 13.487 | 14.593 | 17.837 | 19.766 | 21.523 |

When comparing the tables by giving the stiffness value for translations and rotations, i.e, if it is elastically restrained, the frequency is very much less than classical boundary conditions. Usually in most of the practical cases, the structure may hardly subjected to arbitary boudary conditions. Based on our requirements we can change the value of stiffness of boundary conditions. It is obvious that due to the considered shallow shells are elastic restrained, their displacements at the boundaries are distinct.

## CONCLUSIONS

For any type of shell, ply orientation which gives critical natural frequency may vary with boundary conditions, curvature of shells, other parameters like span to thickness ratios etc, For spherical shallow shell with h/a=1/100, edges are clamped, the ply orientation which gives greater stiffness is [0/90/45/-45/0/90]. But for cylindrical shell with same parameters, the critical one is [0/45/0]. Thus by appropriately orientating the fibres in each lamina of the laminated composite shallow shells, desired strength and stiffness parameters can be achieved.

Keeping parameters like thickness, ply orientation and curvatures constant, clamped conditions improve the natural frequency of shell than simply supported conditions. But always we are not capable to provide classical boundary conditions depending on several factors. The clamped and simply supported boundary conditions cannot be completely realised practically, hence it is essential to access the performance with more realistic boundary conditions. Hence the shell has been analysed using elastically restrained boundary conditions. In elastically restrained boundary conditions, stiffness of each degrees of freedom can be varied according to the realistic conditions. So we adopt elastic boundary condition which is more common than classical boundary conditions today. But frequency may distinct from classical boundary conditions.

For spherical shell, since the ratio of radii of curvature is +1, it takes load effectively than cylindrical. But spherical shell is not suitable for all cases due to the pecularity of places or due to some reasons. When thickness of shell increases, natural frequency also increases. Thus by increasing the thickness of shell comparable with its base dimensions, curvatures etc., we can increase the stiffness of shell to an extent. Thus thickness of shell has considerable effect on its natural frequency.

When radius of curvature to base dimension ratio increases, natural frequency decreases. The reason is when radius of curvature increases without changing the base dimension, the shell is more resembles to plate. Thus its stiffness gets reduced as its inplane load carrying capacity decreases. Thus the shell dynamic characteristics can be improved by optimising the radius of curvature of shell. Spherical shell was effective than cylindrical shell if we are not considering height constraints. i.e, doubly curved shell gives good result than singly curved shell keeping all other parameters constant.

## REFERENCES


1. Book Geoff Eckold, Classical Lamination Theory, Chapter 3, May, 2000, pp. 65-99.
2. Auricchio. F, Sacco. E, Refined First-Order Shear Deformation Theory Models for Composite Laminates, Journal of Applied Mechanics, Vol. 70, May, 2003, pp. 381-390.
3. Rastgaar Aagaah M, Third Order Shear Deformation Theory For Modeling Of Laminated Composite Plates, International Congress and Exposition on Experimental and Applied Mechanics, June, 2004.
4. Reddy J. N., Mechanics of Laminated Composites Plates and Shells, Second Edition, CRC Press, 2004.
5. Seren Akavci, The First Order Shear Deformation Theory For Symmetrically Laminated Composite Plates On Elastic Foundation, The Arabian Journal for Science and Engineering, Vol. 32, June, 2007, pp. 341-348.
6. Alexander Tessler et al, A Consistent Refinement Of First-Order Shear Deformation Theory For Laminated Composite And Sandwich Plates Using Improved Zigzag Kinematics, Journal of Mechanics of Materials and Structures, Vol. 5, February, 2010, pp. 341-361.
7. Zhang X. M, Parametric Analysis of Frequency of Rotating Laminated Composite Cylindrical Shells with the Wave Propagation Approach, Computer methods in Applied Mechanics and Engineering, Vol. 191, March, 2002, pp. 2029-2043.
8. Ribeiro P, Jansen E, Non-Linear Vibrations of Laminated Cylindrical Shallow Shells under Thermo mechanical Loading, Journal of Sound and Vibration, Vol. 315, February, 2008, pp. 626–640.
9. Nguyen-Van et al, Buckling and Vibration Analysis of Laminated Composite Plate/Shell Structures via a Smoothed Quadrilateral Flat Shell Element with In-plane Rotations, Computers and Structures, Vol. 89, February, 2011, pp. 612–625.
10. Mohamad Qatu S, Ebrahim Asadi, Vibration of doubly curved shallow shells with arbitrary boundaries, Applied Acoustics, Vol. 73, July, 2012, pp. 21–27.
11. Tiangui et al, Free Vibration Analysis of Laminated Composite Shallow Shells with General Elastic Boundaries, Composite Structures, Vol. 106, July, 2013, pp. 470-490.
12. Reddy J. N., Laminated Shells, Journals of Engineering Mechanics, Vol. 110, No. 5.
13. Ever Barbero J, Finite Element Analysis of Composite Materials, CFC Press, Taylor and Francis Group, 2011.
14. University of Alberta, Application of Joints and Springs in Ansys, Ansys Tutorials, 2001.
15. Ansys 2004, Ansys Structural Analysis Guide, Release 10.0, Ansys Inc.